

Abstract: The present paper proposes the theoretical study and mathematical approach of laser-induced cavitation microbubble during lithotripsy process in soft tissue. The given model of laser-induced cavitation microbubble via the modified Rayleigh-Plesset equation is analytically solved and investigated via Plesset-Zwick method in order to investigate the behaviour of laser-induced cavitation microbubble and energy produced by laser-induced cavitation microbubble during lithotripsy in soft tissue. From our results, we get the energy produced by laser-induced cavitation bubble increases with increasing of the time and decreasing of the parameter K_{exp} which is calculated experimentally. The present mathematical approach and its analytical results are satisfied by comparing with the published experimentally results.

Introduction: Laser-induced cavitation microbubble during lithotripsy and histotripsy has been widely utilized in the field of biomedical applications [1,2]. The main point is to the utilization of laser-induced cavitation microbubble [3] that contributes to the investigation and improvement of the physical and therapeutic effects which laser-induced cavitation is one of the perspective and effective methods to produce controlled cavitation microbubbles that used to affect and damage the tumor cells during lithotripsy. Noting that, the control of microbubble formation and symmetry is hardly achieved owing to concurrent causes. It is shown, in many technical and biomedical applications, the interaction of cavitation microbubbles in a fluid medium or close to a boundary commonly takes place. Bubble-bubble interaction with the ensuing violent collapse of the bubble, followed by shock wave emission with the formation of a high-speed liquid jet has been linked to erosion damage on solid surfaces in hydraulic machinery as well as some adverse effects on cells and tissue, which exposed to diagnostic and therapeutic ultrasound. In addition, the strength of cavitation has also been exploited in biomedical applications to facilitate targeted drug and gene delivery, improve kidney stone breakup in shock wave lithotripsy, and increase the effectiveness of high-intensity focused ultrasound in cancer therapy. The mathematical approaches of laser-induced cavitation bubble are very important in experimental description in the field of biomedical investigations. Several researchers discussed laser-induced cavitation experimentally where these studies are considered to focus and construction the mathematical approaches as in refs. [4-6]. Up to date, the mathematical models of laser-induced cavitation need to study in a wide range based on the theoretical description in biomedical applications.

The current work aims to investigate the mechanism of laser-induced cavitation microbubble in liquid environments via introducing the theoretical study and mathematical approach based on the radial and dynamics of single microbubble. We focus on the analytical solution of obtained modified Rayleigh-Plesset model of laser-induced cavitation microbubble to evaluate the behaviour of microbubble dynamics and laser energy owing to laser-induced cavitation bubble during lithotripsy.

Physical and mathematical approach: The theoretical model of laser-induced cavitation microbubble is described in Fig. 1 where this method uses to generate the controlled cavitation microbubble that used to affect and damage the tumor cells (as the cells have a stones) during lithotripsy process with considering the physical properties of soft tissue. We suppose that the used time is laser-time which is the time of the behaviour of bubble dynamics during emission of laser beams on the liquid and soft tissue. The geometry of laser-induced cavitation microbubble is assumed to be spherical. The inside of the microbubble contain a uniform pressure of gas.

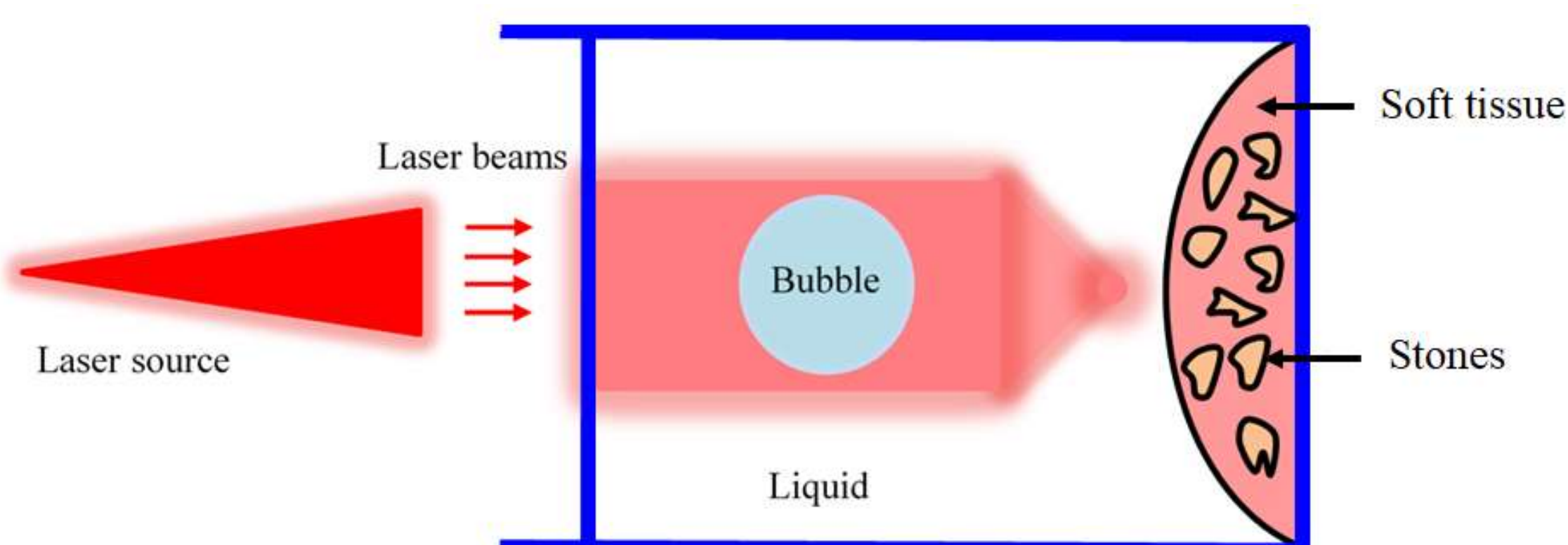


Fig.1 Schematic of laser-induced cavitation microbubble during lithotripsy.

The modified of two-dimensional Rayleigh-Plesset equation (i.e. [6,7]) due to the effect of laser-induced cavitation microbubble can be expressed as

$$R \frac{d^2 R}{dt^2} + \frac{3}{2} \left(\frac{dR}{dt} \right)^2 = \frac{1}{\rho \log\left(\frac{R}{R_\infty}\right)} \left(\Delta P - 4 \frac{\eta \rho}{R} \frac{dR}{dt} - 2 \frac{\sigma}{R} \right) - \frac{1}{2} \left(\frac{1}{2 \log\left(\frac{R}{R_\infty}\right)} - 1 \right) \left(\frac{dR}{dt} \right)^2. \quad (1)$$

Here ΔP is the difference pressure between the pressure $P_b(t)$ within microbubble and ambient pressure P_∞ . η is the dynamic viscosity. σ is the surface tension. R is the radius of microbubble based on laser-induced cavitation.

Initial and boundary conditions of the problem can be put as

$$t = 0, R(0) = R_0, \dot{R}(0) = \dot{R}_0, \ddot{R}(0) = 0. \quad (2)$$

$$t = t_m, R = R_m, \dot{R} \neq 0, \ddot{R} = 0. \quad (3)$$

The laser energy produced laser-induced cavitation microbubble [6] which it defines as

$$E(t) = \int_0^t \Phi(t) dt. \quad \text{And} \quad \Phi(t) = \frac{1}{K_{exp}} \frac{dR}{dt}. \quad (4)$$

Here $\Phi(t)$ is the laser power as a function of time. $\frac{dR}{dt}$ is the laser-induced cavitation microbubble velocity, K_{exp} denotes is constant of correlation [16] which is estimated experimentally and calculated via thermodynamics of superheated vaporization,

Results: We introduce a description analysis of the given results, which illustrates the behaviour of laser-induced cavitation microbubble and the energy produced by laser-induced cavitation microbubble with considering the effect of physical properties in Figs. 2-4 which are plotted in order to analyze and investigate the influence of significant physical parameters such as non-dimension phase transition criteria, energy fluence and parameter K_{exp} .

The comparison between the experimental results [8] and the given theoretical approach of the bubble radius due to laser-induced cavitation with considering the effect of energy fluence is revealed in Fig. 2, where we observed that the calculate the behaviour of laser induced-cavitation microbubble is increasing with the increasing the time; it is called laser-time because the laser-time is time of the behaviour of bubble dynamics during emission of laser beams on the liquid and soft tissue. **To validation our results**, we found that the present theoretical approach agrees with the experimental study [8]; where energy fluence F denotes the energy flow in a laser beam and describes as the energy amount crossing a unit area; $F = \text{energy amount/area}$. The energy produced by laser-induced cavitation microbubble for the different values of physical parameter K_{exp} is shown in Fig 3 where this energy is inversely proportionally to the physical parameter K_{exp} . The impact of Jacob number J_a of non-dimensional of phase transition criteria on the energy produced via laser-induced cavitation microbubble is described in Fig. 4 where the energy power in the function of time and initial void fraction is increasing when non-dimensional of phase transition criteria increases owing to the increasing of initial overheating liquid ΔC_0 . Consequently, an increase in Jacob number J_a of non-dimensional of phase transition criteria increase energy produced via laser-induced cavitation microbubble.

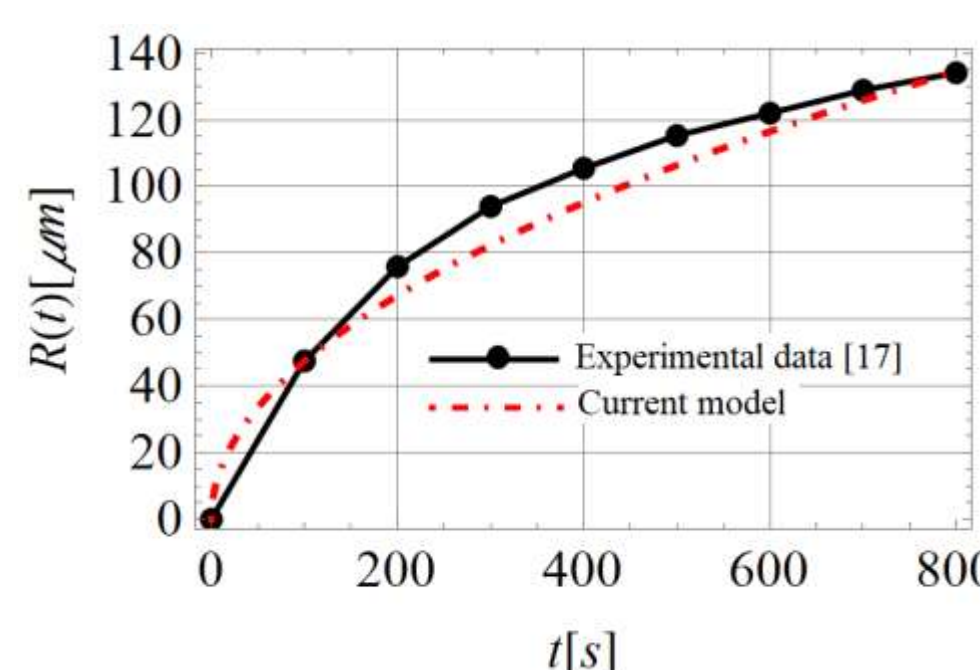


Fig. 2 Comparison of laser-induced cavitation microbubble between the given approach in equation (16) and experimental study in [8]. Energy fluence equals 112.7 J/m^2

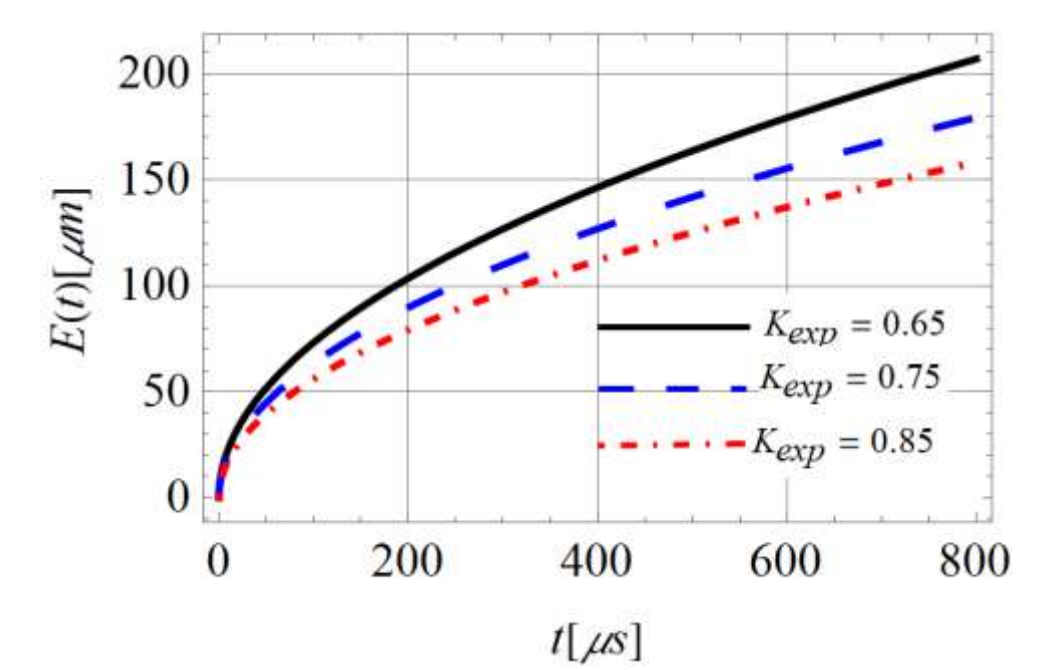


Fig. 3 Energy vs. time produced by laser-induced cavitation microbubble for different values of physical parameter K_{exp}

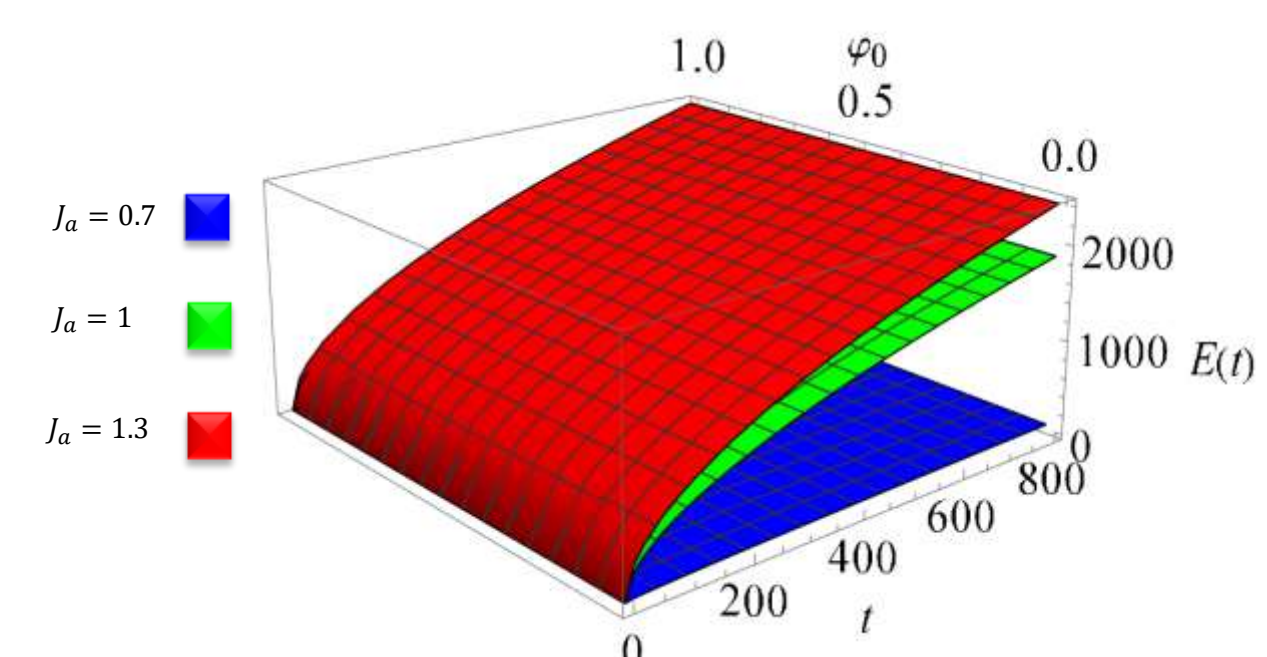


Fig. 4 Evolution of energy power in terms of initial void fraction and time with considering the effect of laser-induced cavitation bubble for different values of Jacob number J_a . under used the values of physical parameters as: $\rho_l = 1050 \text{ kg/m}^3$, $\rho_g = 1.305 \text{ kg/m}^3$, $R_0 = 1 \mu\text{m}$, $R_m = 5 \mu\text{m}$, $\dot{R}_0 = 1 \mu\text{m/s}$, $\eta = 0.075 \text{ Pa s}$, $k_l = 0.613 \text{ W/mk}$, $\sigma = 0.05 \text{ N/m}$, $\Delta C_0 = 1.0 \text{ K}$,

Conclusions: The analytical analysis of the theoretical study and mathematical approach of laser-induced cavitation microbubble is studied in liquid during lithotripsy. The mathematical approach is formulated via the radial and translational dynamics of microbubble. We conclude that the behaviour of laser-induced cavitation microbubble is affected by the laser-time that produced the bubble during lithotripsy. The energy due to laser-induced cavitation microbubble expands slower in increasing of experiment parameter K_{exp} . Dimensionless of phase transition criteria (Jacob number J_a) enhances the energy produced via laser-induced cavitation microbubble. This conclusion should be taken under consideration in the investigations of biomedical applications.

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